SINGLE VERSUS DUAL PROCESS CONTROL CHARTS

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Abstract: Statistical process control (SPC) data obtained from processes to be controlled are often correlated, and therefore invalid for creating a single SPC chart. The single chart is compared with dual common cause and special cause SPC charts. Depending on the application, one or both of these charts is more valuable.

Key words: Economic loss function, Total quality management, Average run length, Moving window spectral method, Stationary.

1. Introduction

Statistical process control (SPC) charts have become a standard tool in manufacturing quality control. Measurements of a product characteristic are plotted on a time chart. The chart is marked with an upper control limit (UCL) and a lower control limit (LCL). The UCL and LCL are based on a specified equal number of standards deviations above and below the mean value of the measurement. The manufacturing process used to make the product is deemed to be in control if all measurements fall within the UCL and LCL. The process is deemed to be out of control if a measurement falls above the UCL or below the LCL. An out of control process is stopped, the cause of the out of control condition is identified and rectified, and the process restarted. The statistical design of the traditional chart is based on the assumption that the measurements are independent of each other, and normally and identically distributed (NID). However, we now know from Alwan and Roberts, 1995, that in about 80 percent of cases, actual process measurements are not NID. There are serially correlated.

Serial correlation in the measurements renders the traditional single SPC chart invalid. This in turn leads to an increased rate of false positive and false negative out of control indications. Various researchers have suggested alternative SPC charts for serially correlated measurements (Box and Jenkins, 1963, Montgomery and Mastrangelo, 1991, Roberts, 1959, Wardel et. al., 1992, 1994, Yashchin, 1987, Crowder, 1987, Hunter, 1986, Lucas and Saccucci, 1990, Ridley and Duke, 2000).

The purpose of this paper is to discuss an alternative to the single chart system, which will be referred to as a dual chart system. The significance and usefulness of the dual chart system will be discussed in the context of traditional manufacturing applications and non traditional applications. In this paper, non traditional applications include but are not limited to biomedical and financial monitoring systems. These were selected because of the widespread need for biomedical monitoring and financial auditing. The rest of the paper is organized as follows. The dual chart system is describe in general in Section 2. Three particular applications are described in Section 3. Concluding remarks are summarized in Section 4.

2. The Dual Chart System

The dual chart system is comprised of what will be referred to as a common cause chart and a special cause chart. The common cause chart contains the component of the data that is systematic (correlated). The special cause chart contains the component of the data that is random (independent). Splitting the data into these two components is accomplished by fitting a time series analysis mathematical model to the data. The mathematical model considered is the Moving Window Spectral (MWS) model and is given in the appendix. The correlation structure of a process variable can be quite complex. The process variable may contain autoregressive processes as well as numerous periodicities. The MWS model is the only model capable of identifying all possible combinations of trend and multiple periodic elements in the data. The common cause chart is constructed from the model fitted values. The special cause chart is constructed from the residual values.

3. Applications

The traditional aggregate historical data, single chart approach to process control is illustrated in Figure A. The dual chart system involving splitting of the data into unique common cause and special cause charts is illustrated in Figures B, C & D.

Manufacturing

Consider the application to an industrial process. Figure B shows the splitting of industrial process measurement data. The data are continuously updated until there is a breach of the special cause chart (SCC), and the process is stopped to prevent the production of defective items. The special cause chart for this type of data may reveal for example, a random one-time raw material departure from what is required. This is an external effect due to a bad batch of raw material. In that case, the supplier must be contacted to rectify the problem.

The common cause chart (CCC) for the type of data expected from a manufacturing process may be a recurring cyclical pattern caused by, for example, a worn machine bearing. This kind of out of control condition is internal to the process. By examining the common cause chart, the pattern can be associated with its probable cause and the defective machine part identified and located. All parts must wear, and must wear out eventually. The defective machine part can be changed during the next scheduled plant maintenance shut down. Otherwise, the bearing wear may reduce product quality. It will fail eventually, forcing an unscheduled shut down, a new machine part anyway, and all the attendant costs of a forced shut down.

Biomedical

Consider the application to monitoring biological vital signs data such as heart rate and oxygen saturation. Unlike a simple machine, the human body has natural internal self-correcting mechanisms. Biological data collected serially over time are therefore related to each other. That is, they are systematic and correlated. They are not independent data. They cannot be represented and analyzed by the traditional single standard control chart because such a chart is valid only for independent data. Even a simple visual analysis would be misleading. Biological data also contain a component that is not correlated. That is, a random and independent component. When the data are represented by the existing method of a single chart, the systematic and random effects are confounded, preventing proper and/or accurate diagnosis of the human biological condition.

Figure C shows the splitting of biological vital signs data. The data are continuously updated regardless of the size of the pulse measurement. Breaches of the special cause chart (SCC) are attributed to one of a kind external environmental special causes, impacting on the body, and that are to be removed if harmful. For example the effects of a one of a kind loud noise that is startling, raises heart rate temporarily but does not repeat in a systematic way. They may also be the effect of a one time application of a medication, the effect of which will eventually work itself into the common cause chart.

Variations on the common cause chart (CCC) are attributed to systematic recurring internal biological common causes that are to be treated accordingly. For example, the appropriate treatment may be by medication. The common cause chart identifies what is systematically occurring, internally, inside the body, due either to normal health, healing or failing health.

Dual charts permit and facilitate proper analysis and diagnosis of the human biological condition as far as can be determined from the data. The dual chart system also reduces the number of false positives (false alarms) that distract the attention of valuable medical personnel away from real and important problems, and reduces the number of false negatives when real and important problems go unnoticed. These features are greatly needed.

Auditing

Consider the application to financial auditing. The common cause chart for this type of data will reflect seasonal and other systematic expenditure patterns that are considered normal. Figure D shows the splitting of accounting cash flow data. The data are continuously updated regardless. That is there is no treatment or action to be taken like in the previous two applications. The common cause chart (CCC) patterns are attributed to systematic changes such as those due to seasonality. Breaches of the special cause chart (SCC) are attributed to one of a kind expenditures, to be investigated and explained. They may be due to legitimate technology expenditures or they may be due to fraud. The special cause chart will indicate unusual expenditures that might otherwise go unnoticed, due their interaction with the systematic effects. They may simply be obscured if the systematic effects are large. The unusual expenditures are marked for further investigation to determine whether they were authorized or unauthorized. Even if the unusual expenditures would have eventually been found by a manual search, the special cause chart saves valuable time and effort.

4. Concluding Remarks

The dual process control chart system described has many advantages over the single process control chart when the data from the process to be controlled, managed or investigated contain systematic variations. How the dual chart system is used depends on the particular application. In manufacturing, the common cause chart can help a quality control engineering to focus on errors in production machinery and equipment. The special cause chart can help a purchasing agent to focus on errors due to a supplier of raw materials. In biomedical monitoring, the common cause chart can help a physician to focus on the internal medication of a patient. The special cause chart can help a nurse to focus on the environmental in which a patient is being cared for. In financial auditing, the common cause chart is not of interest. However, the special cause chart can help an accountant to focus on unusual expenditures that might constitute a fraudulent activity.

APPENDIX
Moving Window Spectral (MWS)
Time Series Model

Consider the process variable, represented as the time series *y(t), t=1,2,...,n*. The time series is assumed to contain trend, constant frequency periodic (cyclical) and random components. In order to estimate the correlation structure of *y(t)*, a moving window of length *T* is defined in the time domain. The moving window is used to generate sequences of data points in the time domain. This creates multiple observations for obtaining least squares estimates of the parameters that describe the behavior of the component cycles over time. The moving window spectral (MWS) time series model is the Tth order autoregressive process given by:

$$y(t) = \sum_{k=1}^{T} y(t-k)b(k) + \varepsilon(t), \qquad t = T+1, T+2, T+3, \ldots, \infty$$

where

b(k) = parameter, coefficient of *y* lagged *k* time periods, $\sum_{k=1}^{T} b(k) < \infty, \qquad \forall T,$

ε(t) = an unobservable error term, sequence of IID normally distributed random variables with mean zero and variance σ^2.

The Fourier transform is used to estimate the spectral density function for each window *y(m-1+t)* from

$$Y_m(\omega) = \sum_{t=1}^{T} y(m-1+t)\exp(-i\omega t)$$

where m=1,2,...,*n*-*T*+1 is the window number, and the index of the realization of a cycle at frequency ω, $-\pi<\omega<\pi$, and $i=\sqrt{-1}$.
Likewise,

$$B(\omega) = \sum_{k=1}^{T} b(k)\exp(-i\omega k)$$

$$\in(\omega) = \sum_{t=1}^{T} \varepsilon(m-1+t)\exp(-i\omega t)\ .$$

The moving window spectral time series model is an integrated procedure of the computer programs *FOURCAST (SPControl*™ , *BioMediControl*™ , *SleepAnalyzer*™ , *Auditor*™) (Ridley, 2001) used in this research.

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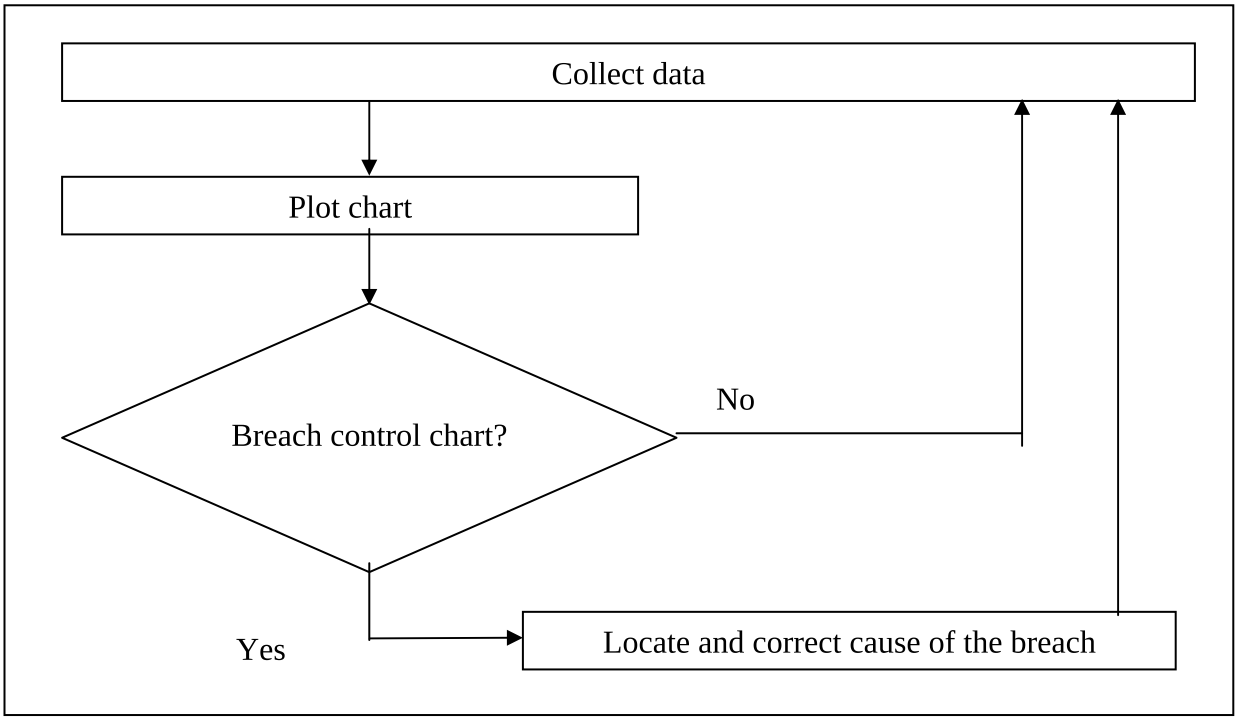

FIGURE A. Traditional single chart process control system.

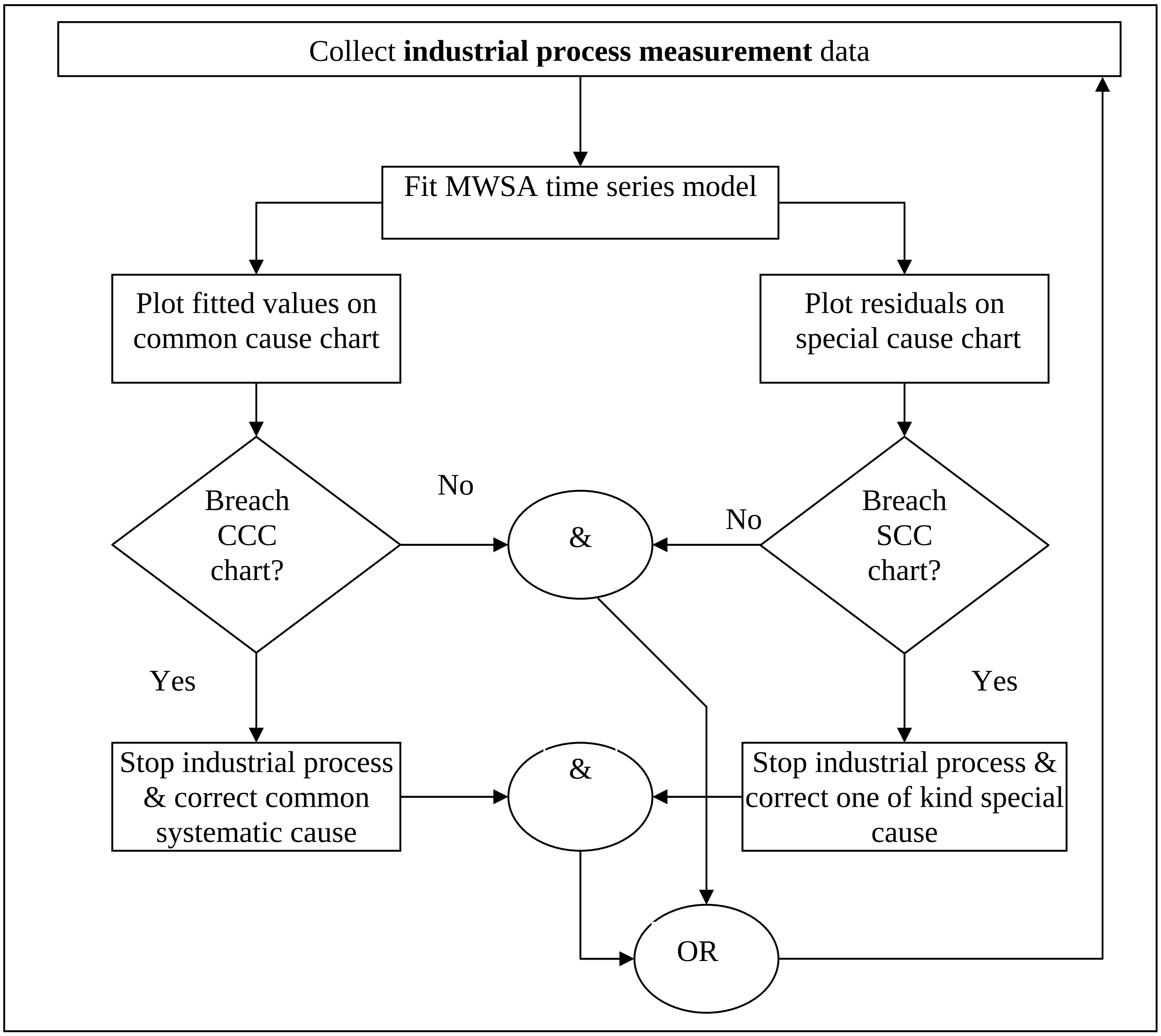

FIGURE B. Dual chart manufacturing process control system.

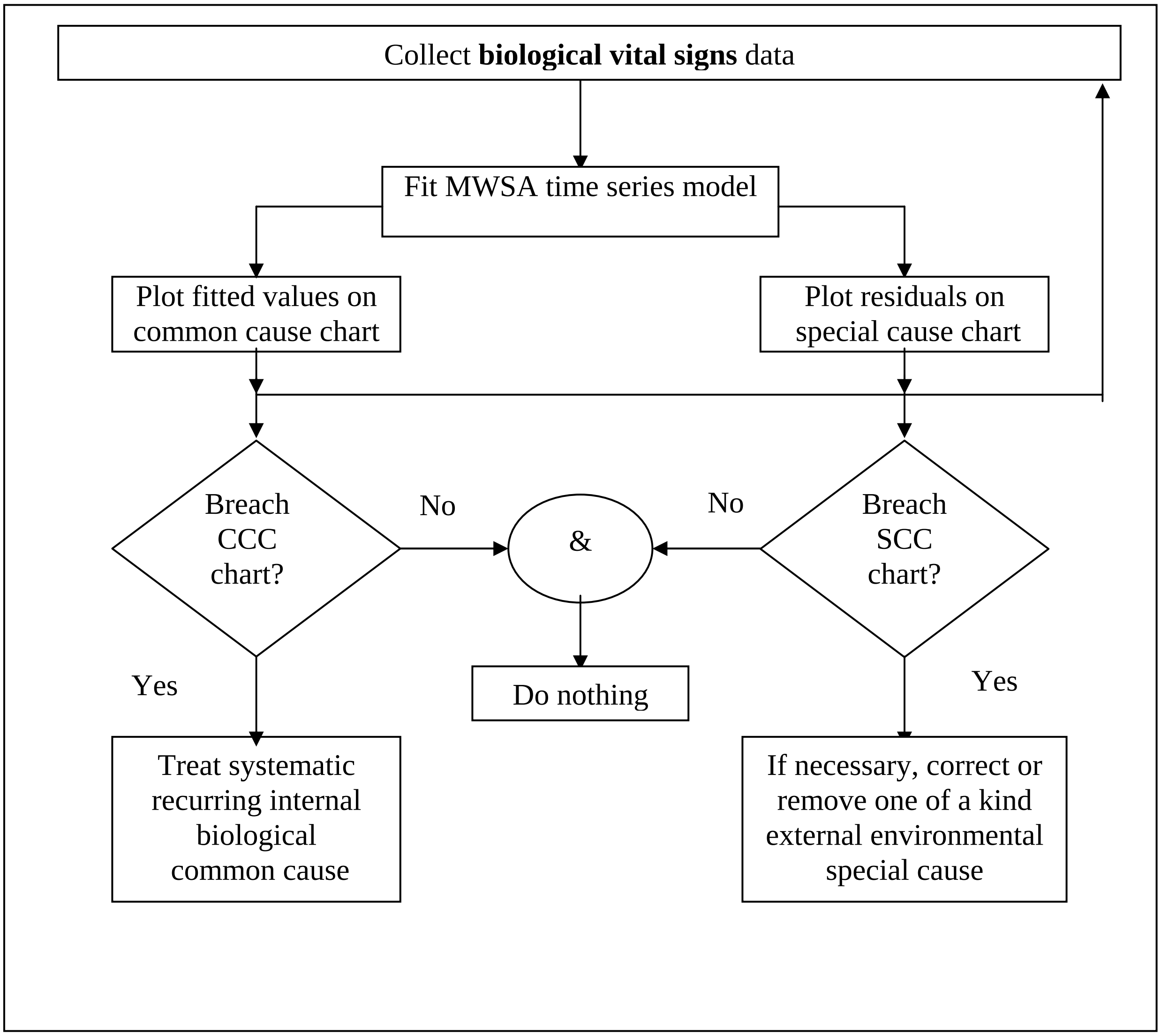

FIGURE C. Dual chart biomedical monitoring system.

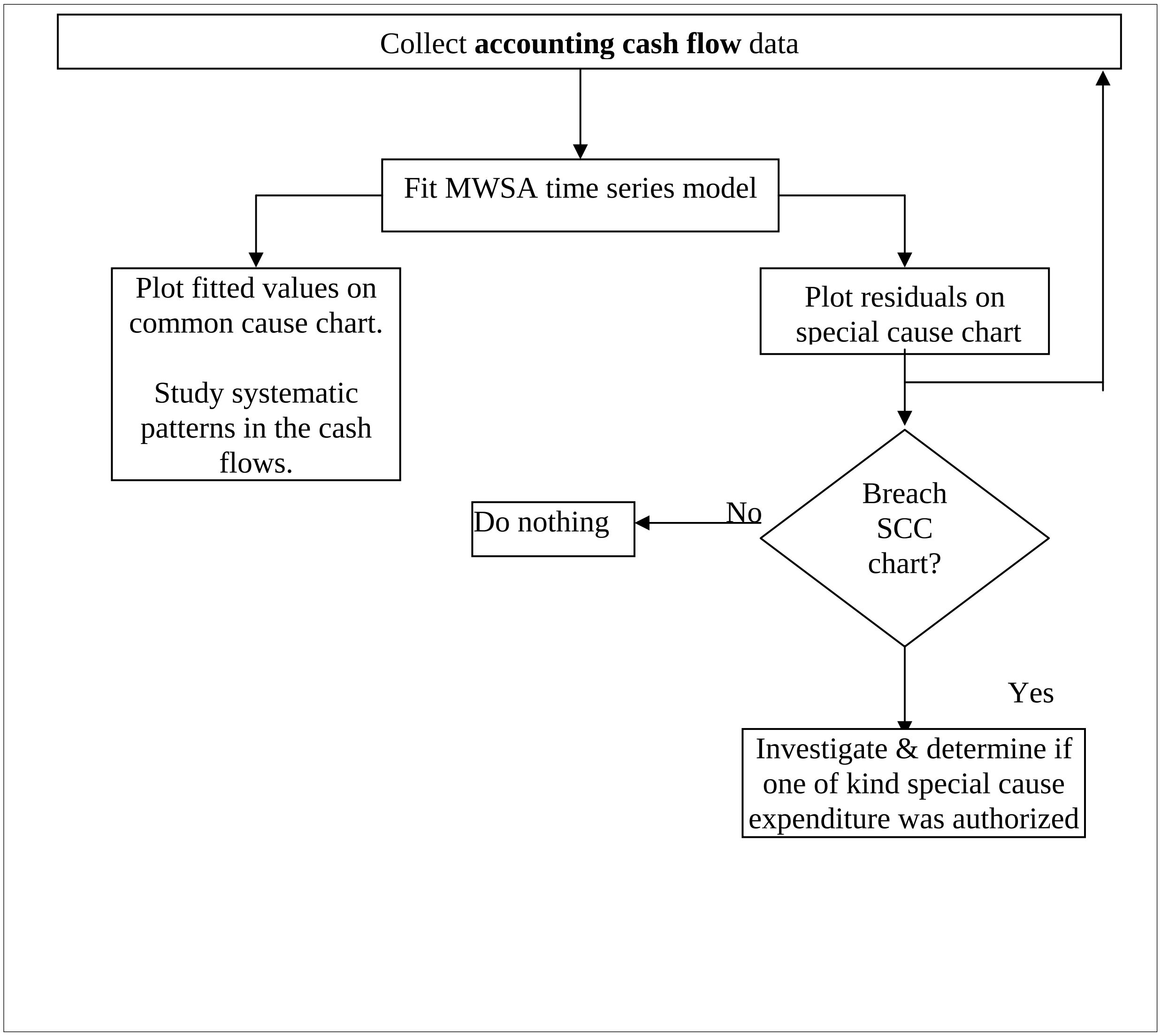

FIGURE D. Dual chart financial auditing system.